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Regression diagnostics in an autocorrelated model

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Abstract: The presence of autocorrelation in a regression model requires the use of the generalized least-squares technique in estimating the regression coefficients. To study the diagnostics of such a model it is therefore necessary to take account of the autocorrelation while re-estimating the parameters after the deletion of an observation. In this paper we look into this problem assuming that the disturbances follow a first-order autoregressive process.

Key words: Autocorrelation, autoregressive process, regression diagnostics.

1 Introduction

In fitting a regression model to a given data set it is vital to know how good the fit is. Use of the model, and particularly predictions based on it, requires that the fitted model is compatible with the data. However, there may often be observations which are different from the others. These outliers frequently have an inordinate influence on least squares estimates. And quite often an outlier may neither show large residuals nor exhibit any influence on the regression line. But outliers need to be detected not only because they may affect the fit but also because they may lead to valuable information regarding the data. To this effect, diagnostics play an important role in regression analysis (see for example Belsley, Kuh and Welsch (1980), Chatterjee and Hadi (1988) or Sen and Srivastava (1990)). These diagnostics provide various ways of studying the residuals and assessing the impact of the respective observations on the regression line. Initially, diagnostic studies used the deletion of observation technique to assess this impact. In recent years Cook's (1986) method for assessing the local influence through model perturbation has also been used by several authors.

However, most of the studies thus far conducted have been restricted to models having uncorrelated disturbances with constant variances. But as has been frequently observed, the dispersion matrix may not be spherical. In such cases it is necessary to apply the generalized least-squares method to estimate the parameters of the model. Hence the usual regression diagnostics need to be modified too. Beach and MacKinnon (1978) circumvent this problem by developing a computationally efficient technique for maximizing the full likelihood function for an autocorrelated linear regression model. In the generalized least-square context, Putterman (1988) studied the influence of the first transformed observation on the parameter estimates. However, Kim amd Huggins (1998) claim that the deletion approach is inappropriate in studying the diagnostics in a regression model with autocorrelated errors. They discuss the effects of simultaneous perturbation of the response vector on all the parameters as also the autocorrelation coefficient. Sharing their concern, Tsai and Wu (1992) used the profile likelihood function to examine the diagnostics through the effects of small perturbations. Schall and Dunne (1991) use a similar technique to study a regression model where the disturbances follow the ARMA model.

The claim of Kim and Huggins (1998) and Tsai and Wu (1992) are only relevant to the point that the deletion of an observation will disrupt the autocorrelation structure. However, with suitable modifications, a transformation matrix similar to that used by Cochrane and Orcutt (1949) can be found that incorporates the deletion of an observation keeping intact the inherent autocorrelation structure. In this paper we seek to find this transformation matrix for a regression model whose disturbances follow a first-order autoregressive model, AR(1), and observe the effect of the deletion of an observation on the generalized least-squares estimates of the regression parameters.

We introduce the model in Section 2, while the main results are discussed in Section 3. Since the estimator will depend on the autoregressive parameter, which is often unknown, in Section 4 we discuss a method of estimating the autocorrelation parameter before using it in the regression estimator. A numerical example is given in Section 5, while some concluding remarks are made in Section 6.

2 The model

Consider the regression model

$$y_t = \mathbf{x}_t\beta + u_t, \qquad t = 1, \ldots, n, \tag{2.1}$$

where y_t is the response at time t, u_t the disturbance term, $\mathbf{x}_t$ the vector of observations on the p explanatory variables and β the px1 parameter vector.

The u_t's are assumed to follow a first-order autoregressive scheme with

$$u_t = \rho u_{t-1} + \epsilon_t, \tag{2.2}$$

where the ϵ_t's are uncorrelated with

$$E(\epsilon_t) = 0, \quad \forall\ t \ \text{ and } \ E(\epsilon_t^2) = \sigma^2, \quad \forall\ t.$$

Expression (2.1) can be written in matrix form as

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{u}, \tag{2.3}$$

where $\mathbf{y}$ and $\mathbf{u}$ are $n \times 1$ vectors and $\mathbf{X}$ is a $n \times p$ matrix.

Using the ordinary least-squares, the parameter β is estimated as

$$\mathbf{b} = (\mathbf{X'X})^{-\mathbf{1}}\mathbf{X'Y}.$$

However, owing to (2.2), $\mathbf{b}$ is not the best linear unbiased estimator (b.l.u.e.) of β.

Since the dispersion of $\mathbf{u}$ is $D(\mathbf{u}) = \sigma^2\Omega$, where

$$\Omega = \frac{1}{(1-\rho^2)} \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ & & \cdots & & \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{bmatrix} \tag{2.4}$$

the b.l.u.e of β is given by

$$\mathbf{b}^* = (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'\Omega^{-1}\mathbf{y}. \tag{2.5}$$

In studying the regression diagnostics it is thus imperative that the residuals considered are not $\mathbf{e} = \mathbf{y} - \mathbf{X}\mathbf{b}$ but $\mathbf{e}^* = \mathbf{y} - \mathbf{X}\mathbf{b}^*$.

Since Ω is a positive definite matrix, there exists a nonsingular matrix $P \ni \Omega^{-1} = P'P$. Therefore, defining $\mathbf{X}^* = P\mathbf{X}$, $\mathbf{y}^* = P\mathbf{y}$ and $\mathbf{u}^* = P\mathbf{u}$, (2.5) can be obtained from the transformed model

$$\mathbf{y}^* = \mathbf{X}^*\beta + \mathbf{u}^* \tag{2.6}$$

as

$$\begin{aligned} \mathbf{b}^* &= (\mathbf{X}'P'P\mathbf{X})^{-1}\mathbf{X}'P'P\mathbf{y} \\ &= (\mathbf{X}^{*'}\mathbf{X}^*)^{-1}\mathbf{X}^{*'}\mathbf{y}^*. \end{aligned} \tag{2.7}$$

Hence when the diagnostic relates to the deletion of one observation at a time and observing the discrepancy it produces, the corresponding row and column of Ω must be deleted. This would mean that the corresponding column of P is deleted. Here we study the effect of this on the model (2.1)-(2.2).

3 The main results

The dispersion matrix Ω in (2.4) has the inverse

$$\Omega^{-1} = \begin{bmatrix} 1 & -\rho & 0 & \cdots & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \cdots & 0 & 0 \\ & & \cdots & & & \\ 0 & 0 & 0 & \cdots & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix}$$

which can be also be written uniquely as $P^{'}P$ (see, for example, Judge et al (1988), pp 389), where

$$P = \begin{bmatrix} \sqrt{1-\rho^2} & 0 & 0 & \dots & 0 & 0 \\ -\rho & 1 & 0 & \dots & 0 & 0 \\ 0 & -\rho & 1 & \dots & 0 & 0 \\ & & \dots & & & \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & -\rho & 1 \end{bmatrix}.$$

Now suppose the ith observation is deleted. Then the ith row and column of Ω also needs to be deleted. To observe the effect of this deletion on Ω^{-1} and P, let Ω_i denote the matrix obtained from Ω by deleting the ith row and ith column. Let, for the sake of simplicity, the (n-1) rows and columns of Ω_i, Ω_i^{-1} and the corresponding P_i be denoted by $1, \dots, (i-1), (i+1), \dots, n$ respectively.

Result 3.1. *For $i = 2, \dots, (n-1), \Omega_i^{-1}$ is obtained from Ω^{-1} by deleting the ith row and ith column and replacing the (i-1, i-1) and (i+1, i+1) elements by $1 + \rho^4/(1+\rho^2)$, and the (i-1, i+1) and (i+1, i-1) elements by $-\rho^2/(1+\rho^2)$. The corresponding P_i is obtained from P by deleting the ith row and ith column and replacing the (i+1, i-1) element by $-\rho^2/(1+\rho^2)^{1/2}$ and the (i+1, i+1) element by $1/(1+\rho^2)^{1/2}$.*

Note. For $i = 1$ and n, Ω_i will be of the same form as Ω except for a single reduction in the dimension. Hence the corresponding Ω_i^{-1} and P_i will be the same as Ω^{-1} and P with one dimension less.

Proof. The proof follows by partitioning

$$\Omega_i = \frac{1}{1-\rho^2} \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{i-2} & \rho^i & \dots & \rho^{n-1} \\ \rho & 1 & \rho & \dots & \rho^{i-3} & \rho^{i-1} & \dots & \rho^{n-2} \\ & & & \dots & & & & \\ \rho^{i-2} & \rho^{i-3} & \rho^{i-4} & \dots & 1 & \rho^2 & \dots & \rho^{n-i+1} \\ \rho^i & \rho^{i-1} & \rho^{i-2} & \dots & \rho^2 & 1 & \dots & \rho^{n-i-1} \\ & & & \dots & & & & \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \dots & \rho^{n-i+1} & \rho^{n-i-1} & \dots & 1 \end{bmatrix}$$

as $\Omega_i = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, where A_{11} and A_{22} are square matrices of order i-1 and n-i respectively, and then inverting the partitioned matrix using results from Rao (1973). ∇

Let $\mathbf{X}_i, \mathbf{y}_i$ and $\mathbf{u}_i$ be $\mathbf{X}, \mathbf{y}$ and $\mathbf{u}$ with the ith observation deleted. Then defining $\mathbf{X}_i^* = P_i\mathbf{X}_i$ and $\mathbf{y}_i^* = P_i\mathbf{y}_i$, the estimator of β after the deletion of the

ith observation can be obtained as

$$\begin{aligned}\mathbf{b}_i^* &= (\mathbf{X}_i^{'}\Omega_i^{-1}\mathbf{X}_i)^{-1}\mathbf{X}_i^{'}\Omega_i^{-1}\mathbf{y}_i \\ &= (\mathbf{X}_i^{'}P_i^{'}P_i\mathbf{X}_i)^{-1}\mathbf{X}_i^{'}P_i^{'}P_i\mathbf{y}_i \\ &= (\mathbf{X}_i^{*'}\mathbf{X}_i^*)^{-1}\mathbf{X}_i^{*'}\mathbf{y}_i^*. \end{aligned} \tag{3.1}$$

For each of the (n-2) intermediate observations it is however not necessary to calculate $\mathbf{b}_i^*$ afresh. This can more easily be obtained from the relationship between $\mathbf{b}^*$ and $\mathbf{b}_i^*$.

Result 3.2. *Let $\mathbf{x}_i^*$ denote the ith row of $\mathbf{X}^*$ and y_i^* the ith element of $\mathbf{y}^*$. Then for $i = 2, \ldots, (n-2)$,*

$$\mathbf{b}_i^* = \mathbf{b}^* - [(\rho\epsilon_{i+1}^* - \epsilon_i^*)/(1+\rho^2)^{1/2}(1+\mathbf{u}_i^{*'}(\mathbf{X}^{*'}\mathbf{X}^*)^{-1}\mathbf{u}_i^*)](\mathbf{X}^{*'}\mathbf{X}^*)^{-1}\mathbf{u}_i^*, \tag{3.2}$$

where $\mathbf{u}_i^* = (1+\rho^2)^{1/2}(\rho\mathbf{x}_{i+1}^* - \mathbf{x}_i^*)$ and $\epsilon_i^* = y_i^* - \mathbf{b}^{*'}\mathbf{x}_i^*$.

Proof. Let $\mathbf{x}_{i(j)}^*$ denote the jth row of $\mathbf{X}_i^*$. Then noting that $\mathbf{x}_{i(i)}^*$ is missing, $\mathbf{x}_{i(j)}^* = \mathbf{x}_j^*$ for $j = 1, \ldots, (i-1), (i+2), \ldots, n$ and only $\mathbf{x}_{i(i+1)}^*$ is different from $\mathbf{x}_{i+1}^*$,

$$\mathbf{X}^{*'}\mathbf{X}^* = \sum_{i=1}^{n}\mathbf{x}_i^{*'}\mathbf{x}_i^*$$

and

$$\begin{aligned}\mathbf{X}_i^{*'}\mathbf{X}_i^* &= \sum_{j=1,\neq i}^{n}\mathbf{x}_{i(j)}^{*'}\mathbf{x}_{i(j)}^* \\ &= \sum_{j=1}^{n}\mathbf{x}_j^{*'}\mathbf{x}_j^* - \mathbf{x}_i^{*'}\mathbf{x}_i^* - \mathbf{x}_{i+1}^{*'}\mathbf{x}_{i+1}^* + \mathbf{x}_{i(i+1)}^{*'}\mathbf{x}_{i(i+1)}^* \\ &= \mathbf{X}^{*'}\mathbf{X}^* - \mathbf{u}_i^{*'}\mathbf{u}_i^*. \end{aligned} \tag{3.3}$$

Similarly,

$$\mathbf{X}_i^{*'}\mathbf{y}_i^* = \mathbf{X}^{*'}\mathbf{y}^* - \mathbf{u}_i^{*'}\mathbf{v}_i^* \tag{3.4}$$

where $v_i^* = (1+\rho^2)^{1/2}(\rho y_{i+1}^* - y_i^*)$.
Combining (3.3) and (3.4) and then using simple algebra, (3.2) follows. ∇

Result 3.3. *Under the above set-up,*

(i) $\mathbf{b}_i^*$ *is the best linear unbiased estimator of* β *and*

(ii) If $n^{-1}(\mathbf{X}'\Omega^{-1}\mathbf{X}) \to \Sigma$, *where* Σ *is a p.d. matrix of constants,* $\mathbf{b}_i^*$ *is consistent for* β.

Note. $n^{-1}(\mathbf{X}'\Omega^{-1}\mathbf{X}) \to \Sigma$ implies that $\mathbf{b}^*$ is consistent for β.

Proof. For the model,

$$\mathbf{y}_i = \mathbf{X}_i\beta + \mathbf{u}_i,$$

since $E(P_i\mathbf{u}_i) = 0$ and $D(P_i\mathbf{u}_i) = \sigma^2 I_n$, by the Gauss-Markov Theorem, $\mathbf{b}_i^*$ is the best linear unbiased estimator of β. Also $(n-1)^{-1}\mathbf{X}_i^{*'}\mathbf{X}_i^* = (n-1)^{-1}\mathbf{X}^{*'}\mathbf{X}^* - (n-1)^{-1}\mathbf{u}_i^{*'}\mathbf{u}_i^* \to \Sigma$, so that $\mathbf{b}_i^*$ is consistent for β. ∇

4 Unknown ρ

Observe that $\mathbf{b}_i^*$ depends on the parameter ρ through Ω_i ((3.1) or (3.2)), so that ρ needs to be known to calculate it. However, in many situations ρ may not be known. In such cases it is required to estimate it before calculating $\mathbf{b}_i^*$. In the usual situation of regression with autocorrelated errors, Prais and Winsten (1954) suggest using the transformation matrix P and rewriting the model as

$$y_t - \mathbf{x}_t\beta = \rho(y_{t-1} - \mathbf{x}_{t-1}\beta) + \epsilon_t \tag{4.1}$$

to get an estimate of ρ through the ordinary least squares technique by substituting $\mathbf{b}$ for β. This actually leads to the estimator r of ρ as

$$r = (\sum_{t=2}^{n} e_t e_{t-1})/(\sum_{t=2}^{n} e_{t-1}^2), \tag{4.2}$$

where e_t is the ith element of $\mathbf{e}$.

However, when the ith observation is deleted and P_i is used instead of P, (4.1) is valid only for the n-3 terms $t = 2, \ldots, i-1$ and $i+2, \ldots, n$, since these rows of P_i are identical to those of P. The $(i+1)$th row of P_i will lead to the transformation

$$y_{i+1} - \mathbf{x}_{i+1}\beta = \rho^2(y_{i-1} - \mathbf{x}_{i-1}\beta) + (\rho\epsilon_i + \epsilon_{i+1}). \tag{4.3}$$

Obviously the ordinary least squares method to estimate ρ is no longer tenable if we include (4.3). Noting the fact that the error variance in (4.3) is $\sigma^2(1+\rho^2)$, a weighted least squares method, with the inverses of the error variances as weights, can be applied. However, this will lead to non-linear normal equations which will require some iterative technique to be solved. A simpler method to obtain an estimate of ρ would be to ignore the $(i+1)$th term and consider only the $n-3$ terms $t = 2, \ldots, i-1$ and $i+2, \ldots, n$.

Here it would also be appropriate to substitute $\mathbf{b}_i$, the least squares estimator of β obtained after deleting the ith observation, and not $\mathbf{b}$, for β. Defining the leverages

$$h_{ii} = \mathbf{x}_i^{'}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_i,$$

$\mathbf{b}_i$ can be written in terms of $\mathbf{b}$ as

$$\begin{aligned}\mathbf{b}_i &= (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{y}_i \\ &= \mathbf{b} - (1-h_{ii})^{-1}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_i e_i. \end{aligned} \tag{4.4}$$

Thus writing $e_{it} = y_t - \mathbf{x}_t'\mathbf{b}_i$, an estimator r_i^* of ρ can be defined as

$$r_i^* = \sum_{t=2,\neq i,i+1}^{n} e_{it}e_{i(t-1)} \Big/ \sum_{t=2,\neq i,i+1}^{n} e_{i(t-1)}^2. \tag{4.5}$$

Note that using (4.4),

$$\begin{aligned} e_{it} &= e_t + (1-h_{ii})^{-1}\mathbf{x}_t'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_i e_i \\ &= e_t + (1-h_{ii})^{-1}h_{it}e_i \end{aligned} \tag{4.6}$$

i.e. the e_{it}'s can be expressed solely in terms of the residuals and leverages of the original undeleted set of observations.

Defining $\hat{\Omega}_i$ as Ω_i with ρ replaced by r_i^*, the estimated generalized least squares estimator of β after deleting the ith observation is

$$\hat{\mathbf{b}}_i^* = (\mathbf{X}_i'\hat{\Omega}_i^{-1}\mathbf{X}_i)^{-1}\mathbf{X}_i'\hat{\Omega}_i^{-1}\mathbf{y}_i. \tag{4.7}$$

However, since it would be logical to use r in place of ρ in $\mathbf{b}^*$ to get $\hat{\mathbf{b}}^*$, a relationship between $\hat{\mathbf{b}}_i^*$ and $\hat{\mathbf{b}}^*$ of the type (3.2) would be difficult to find. Since like r, r_i^* would also be a consistent estimator of ρ, the conditions (see Judge et.al., 1988) that $\hat{\mathbf{b}}_i^*$ is asymptotically equivalent to $\mathbf{b}_i$ are satisfied so that $\hat{\mathbf{b}}_i^*$ is also a consistent estimator for β.

5 A numerical example

Here we cite an example to illustrate our method. Let $p = 1$ and suppose for a set of x's, the y's are generated as

$$\begin{aligned} y_i &= 1 + 0.9x_i + u_i, \\ u_i &= 0.8u_{i-1} + \epsilon_i, \end{aligned}$$

where the ϵ_i's are independently normally distributed with mean 0 and variance 1. (The x's and the corresponding y's thus obtained are shown in Table 5.1.) Of the 20 observations in the data, the 14th observation has a value of x which is much smaller than the other 19. The regression of y on x using the ordinary least squares gives the following estimated regression line,

$$y = 0.07 + 0.737x,$$

with Durbin-Watson $d = 0.73$, which implies a positive autocorrelation.

Assuming that the value of ρ is unknown, the comparisons are made through the following $DFBETA$ and $DFFIT$ values defined as

$$DFBETA_i = \mathbf{b}^* - \hat{\mathbf{b}}_i^*, \quad \text{and} \quad DFFIT_i = (\mathbf{b}^* - \hat{\mathbf{b}}_i^*)\mathbf{x}_i = y_i^* - y^{(i)*}.$$

Note. If ρ is known then the $DFBETA$ and $DFFIT$ are to be defined as $DFBETA_i = \mathbf{b}^* - \mathbf{b}_i^*$, and $DFFIT_i = (\mathbf{b}^* - \mathbf{b}_i^*)\mathbf{x}_i$.

It is often appropriate to standardize the $DFBETA$ and $DFFIT$ values by the corresponding standard deviations (Neter, Wasserman & Kutner, 1985). Since the dispersion matrix of $\mathbf{b}^*$ is $\sigma^2(\mathbf{X}^{*'}\mathbf{X}^*)^{-1}$, the standardizing factor for the jth element of the $DFBETA_i$ should be $\sigma a_{jj}^{*\,\frac{1}{2}}$, where a_{jj}^* is the jth diagonal element of $(\mathbf{X}^{*'}\mathbf{X}^*)^{-1}$, with ρ, if unknown, being replaced by r as in (4.2). However, since the σ^2 will in general be unknown, it needs to be estimated. Let

$$s_{(i)}^{*2} = \frac{1}{n-p-3} \sum_{j=1 \neq i, i+1}^{n} (y_j - \mathbf{x}_j^{'}\hat{\mathbf{b}}_i^*)^2.$$

Then $s_{(i)}^{*2}$ can be used as the estimator of σ^2 in $DFBETA_i$. The corresponding $DFFIT$s are standardized by $s_{(i)} h_{ii}^{*\frac{1}{2}}$, where $h_{ii}^* = \mathbf{x}_i^{'}(\mathbf{X}^{*'}\mathbf{X}^*)^{-1}\mathbf{x}_i$.

Table 1 shows the standarized $DFBETA$ and standardized $DFFIT$ values corresponding to the 2nd to the 19th observations (Note that here $DFBETA$ is a 2-element vector giving the intercept and the slope respectively). The values corresponding to the 1st and 20th observations are not shown since the results are the same for our technique as for the usual method. As can be observed from the Table, the 14th observation, which is an outlier in x, shows significantly high values of both the standardized $DFBETA$ values of the intercept and the slope as also the standardized $DFFIT$ value.

6 Concluding remarks

Our results suggest that the deletion technique for detecting outliers can be easily extended to models with AR(1) errors. Some complications do come in if the autoregressive parameter is unknown, but even then it is simpler to use than the perturbation technique.

The technique can, with the necessary modifications, be extended to AR(p) models with $p > 1$, as also to moving average (MA) and autoregressive moving average (ARMA) error models. In each of these cases the key would be to invert the corresponding error dispersion matrix (see Judge et.al., 1988) and then obtain a suitable transformation matrix P^* after deleting a row. These would obviously be more complex than for the AR(1) case, but should be interesting to look at. Nevertheless the AR(1) error structure is the one most widely used in regressions with autocorrelated errors and hence the simple nature of the results of this paper should be useful to practitioners for detecting outliers.

Table 1 *The standardized $DFBETA$ and $DFFIT$ values.*

X	Y	$DFBETA$	$DFFIT$
8.34633	9.09170	-	-
7.38356	7.82920	-0.29495, -0.13761	-1.1486
8.84329	9.28896	-0.32819, 0.04530	-0.7610
7.31505	7.85355	-0.03724, -0.16186	-0.4595
8.93722	7.30350	-0.04991, 0.06706	0.0320
5.04525	3.31072	-0.17935, 0.31346	-0.0423
7.53255	4.85930	0.60098, -0.01508	1.6841
4.79261	1.34335	-0.47724, 0.88895	-0.0961
9.06422	5.38780	0.22451, 0.22251	1.1293
8.58329	5.42496	0.62729, 0.08802	1.9098
3.33611	0.32250	-0.51927, 1.00080	-0.4005
6.31859	3.55673	0.46661, 0.05369	1.4442
7.22386	4.05147	-0.07446, -0.79190	-1.9207
0.09308	3.30623	4.26570, -5.24860	6.2489
9.38879	7.18991	0.36371, -0.61954	-0.6035
2.44619	1.46157	-0.74564, 0.91831	-1.0598
6.85166	5.06650	0.26695, 0.11586	1.0096
7.99380	4.84442	0.08387, 0.03794	0.3234
9.25860	6.00274	0.32308, 0.08664	1.0400
6.00619	3.29558	-	-

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