

Binary data in the presence of covariates and misclassifications: a Bayesian approach

Jorge Alberto Achcar^{1,2}, Edson Zangiacomi Martinez¹

and Francisco Louzada-Neto²

¹ *Universidade de São Paulo*

² *Universidade Federal de São Carlos*

Abstract: In this paper we introduce a Bayesian analysis for binary data in the presence of covariates and misclassifications. As a special situation in diagnostic medical testing, we obtain Bayesian inferences for the sensitivity and the specificity in the presence of covariates. We consider a situation where the individuals can be verified or unverified about their real disease status after a test. When part or even all individuals are not verified, usually we have great difficulties to get classical inference results for the parameters of interest. For this situation, the introduction of latent variables gives a good alternative to deal with missing data under the Bayesian approach, specially using Markov chain monte Carlo (MCMC) methods to obtain the posterior summaries of interest. We illustrate the proposed methodology on three real data sets.

Key words: Bayesian analysis, binary data, MCMC methods, sensitivity and specificity.

1 Introduction

In many applications of binary data, we have the presence of covariates and misclassifications. Interested readers can refer to Geng and Asano (1989), Evans *et al.* (1996), Rekaya *et al.* (2001) and Soares and Paulino (2001) which are very informative papers on this area. As a special situation, we could have the presence of false positives or false negatives in medical diagnostic testing, where we denote the sensitivity as the probability of a positive test given that the patient really has the disease and the specificity as the probability of a negative test given that the patient do not have the disease (see, for example, Pepe, 2003). These probabilities are of great medical interest.

The use of Bayesian methods has been considered by many authors to analyse binary response data (see, for example, Dey *et al.*, 2000) as an alternative for the usual classical approach under which inferences are based on asymptotic theory. Under the Bayesian paradigm it is natural to deal with more problematic cases, such as misclassification, incomplete data or measurement errors, as it is the case

of misclassified categorical data (see, for example, Paulino *et al.*, 2003; Rekaya *et al.*, 2001 or Soares and Paulino, 2001)

For studies of the performance of new medical tests, we also could have other situations of missing data when part or even all individuals are not verified by a "gold standard". In this case, if we consider only the verified individuals (see, for example, Begg, 1987; Begg and Greenes, 1983 or Zhou, 1993) we could obtain biased estimators for the sensitivity and the specificity of the test.

Let p be the prevalence of disease in the population and D the disease status, where $D = 1$ (or simply, D) denotes an individual with the disease and $D = 0$ (or \bar{D}) denotes a free-disease individual. Thus, $p = P(D)$. Let T be a random variable related to the diagnostic test results, where $T = 1$ (or T) denotes a positive test (success) and $T = 0$ (or \bar{T}) denotes a negative test. The sensitivity of the diagnostic test is given by $S_E = P(T|D)$ and the specificity is given by $S_P = P(\bar{T}|\bar{D})$. Observe that $1 - S_E$ and $1 - S_P$ are the probabilities of misclassifications.

In the presence of a vector of covariates $\mathbf{X}'_i = (X_{0i}, X_{1i}, \dots, X_{Li})$, the probability of a positive test is given by

$$\eta_i = p_i S_E + (1 - p_i)(1 - S_P) \quad (1.1)$$

where $i = 1, \dots, n$. In the same way, the probability of a negative test is given by

$$1 - \eta_i = p_i (1 - S_E) + (1 - p_i) S_P.$$

Different parametric choices for p_i could be considered. We however assume a logistic regression model given by

$$p_i = \frac{\exp\left(\sum_{j=0}^L \beta_{pj} x_{ji}\right)}{1 + \exp\left(\sum_{j=0}^L \beta_{pj} x_{ji}\right)}, \quad (1.2)$$

where p_i is the prevalence of the disease for an individual with covariate \mathbf{X}_i , $X_{0i} = 1$, $i = 1, \dots, n$. Observe that we could also assume S_E and S_P dependent on the covariates, but this situation is out of the scope of the paper.

In this paper, we introduce a Bayesian analysis for misclassification models in the presence of covariates using Markov chain Monte Carlo (MCMC) methods (see, for example, Gelfand and Smith, 1990). In Section 2, we develop a Bayesian analysis assuming that all individuals are unverified about the real disease status after a test. In Sections 3 and 4, we consider the case where a proportion of the individuals are verified while others are unverified for their real disease status. In Section 5 the proposed methodology is illustrated on two real medical data.

2 A Bayesian analysis assuming all individuals are unverified

Let us assume that all individuals are unverified about the real disease status after an application of a medical test. Assuming a Bernoulli distribution for the test result t_i with success probability given by (1.1), let $D_i = (t_i, \mathbf{x}_i)$, $i = 1, \dots, n$ be the data where $t_i = 1$ (T) or 0 (\bar{T}) and \mathbf{X}_i is a covariate vector associated to each individual. Also assume the logit link (1.2) for the probability of prevalence of disease for the individual in the population. The likelihood function for $\theta' = (S_E, S_P, \beta_{p0}, \beta_{p1}, \dots, \beta_{pL})$ is given by

$$L(\theta) = \prod_{i=1}^n [p_i S_E + (1 - p_i)(1 - S_P)]^{t_i} [p_i(1 - S_E) + (1 - p_i)S_P]^{1-t_i}. \quad (2.1)$$

For a Bayesian analysis of the model let us assume the following prior distributions for S_E, S_P and $\beta'_p = (\beta_{p0}, \beta_{p1}, \dots, \beta_{pL})$:

$$\begin{aligned} S_E &\sim \text{Beta}(a, b); \quad a, b \text{ known;} \\ S_P &\sim \text{Beta}(c, d); \quad c, d \text{ known;} \\ \beta_{pj} &\sim N(e_j; f_j^2); \quad e_j, f_j \text{ known,} \end{aligned} \quad (2.2)$$

where $j = 0, 1, \dots, L$, $\text{Beta}(a, b)$ denotes a Beta distribution with mean $a/(a+b)$ and variance $ab/[(a+b)^2(a+b+1)]$ and $N(\mu, \sigma^2)$ denotes a normal distribution with mean μ and variance σ^2 . We further assume prior independence among the parameters.

Following Tanner and Wong (1987), to obtain better performance for the Gibbs sampling algorithm, we introduce the latent variables D_{ik}^* (disease status) given $T_i = |k-2|$ for $k = 1, 2$ with a Bernoulli distribution with success probability h_{ki} given by

$$h_{1i} = P(D_{i1}^* | T_i) = \frac{p_i S_E}{p_i S_E + (1 - p_i)(1 - S_P)}, \quad (2.3)$$

for $k = 1$, and

$$h_{2i} = P(D_{i2}^* | \bar{T}_i) = \frac{p_i(1 - S_E)}{p_i(1 - S_E) + (1 - p_i)S_P}, \quad (2.4)$$

for $k = 2$. Thus, the joint distribution for D_{i1}^* , $i = 1, \dots, n$, is given by

$$\begin{aligned} f(d_{11}^*, \dots, d_{n1}^*) &\propto \prod_{i=1}^n h_{1i}^{t_i d_{i1}^*} (1 - h_{1i})^{t_i(1-d_{i1}^*)} \\ &\propto \prod_{i=1}^n \frac{(p_i S_E)^{t_i d_{i1}^*} [(1 - p_i)(1 - S_P)]^{t_i(1-d_{i1}^*)}}{[p_i S_E + (1 - p_i)(1 - S_P)]^{t_i}}, \end{aligned} \quad (2.5)$$

and the joint distribution for D_{i2}^* , $i = 1, \dots, n$ is given by

$$\begin{aligned} f(d_{12}^*, \dots, d_{n2}^*) &\propto \prod_{i=1}^n h_{2i}^{(1-t_i)d_{i2}^*} (1-h_{2i})^{(1-t_i)(1-d_{i2}^*)} \\ &\propto \prod_{i=1}^n \frac{[p_i(1-S_E)]^{(1-t_i)d_{i2}^*} [(1-p_i)S_P]^{(1-t_i)(1-d_{i2}^*)}}{[p_i(1-S_E) + (1-p_i)S_P]^{1-t_i}}. \end{aligned} \quad (2.6)$$

Combining (2.5) and (2.6) with (2.1), and assuming the prior distributions (2.2), the joint posterior distribution for θ and \mathbf{d}^* is given by

$$\begin{aligned} \Pi(\theta, \mathbf{d}^* | \mathbf{t}) &\propto S_E^{a+\sum_{i=1}^n t_i d_{i1}^* - 1} (1-S_E)^{b+\sum_{i=1}^n (1-t_i)d_{i2}^* - 1} S_P^{c+\sum_{i=1}^n (1-t_i)(1-d_{i2}^*) - 1} \\ &\quad \times (1-S_P)^{d+\sum_{i=1}^n t_i(1-d_{i1}^*) - 1} \prod_{j=0}^L \exp\left[-\frac{1}{2f_j^2} (\beta_{pj} - e_j)^2\right] \\ &\quad \times \prod_{i=1}^n p_i^{t_i d_{i1}^* + (1-t_i)d_{i2}^*} (1-p_i)^{t_i(1-d_{i1}^*) + (1-t_i)(1-d_{i2}^*)}. \end{aligned} \quad (2.7)$$

The conditional distributions for the Gibbs sampling algorithm are given by

$$\begin{aligned} \text{(a)} \quad S_E | \theta_{(S_E)}, \mathbf{t}, \mathbf{d}^* &\sim \text{Beta}\left(a + \sum_{i=1}^n d_{i1}^* t_i, b + \sum_{i=1}^n d_{i2}^* (1-t_i)\right), \\ \text{(b)} \quad S_P | \theta_{(S_P)}, \mathbf{t}, \mathbf{d}^* &\sim \text{Beta}\left(c + \sum_{i=1}^n (1-d_{i2}^*) (1-t_i), d + \sum_{i=1}^n t_i (1-d_{i1}^*)\right), \\ \text{(c)} \quad \Pi(\beta_{pj} | \theta_{(\beta_{pj})}, \mathbf{t}, \mathbf{d}^*) &\propto \exp\left[-\frac{1}{2f_j^2} (\beta_{pj} - e_j)^2\right] \psi(\theta), \end{aligned} \quad (2.8)$$

where

$$\psi(\theta) = \exp\left\{\beta_{pj} \sum_{i=1}^n x_{ji} [t_i d_{i1}^* + (1-t_i) d_{i2}^*] - \sum_{i=1}^n \ln\left[1 + \exp\left(\sum_{j=0}^L \beta_{pj} x_{ji}\right)\right]\right\},$$

$x_{0i} = 1$, $i = 1, \dots, n$ and $\theta_{(v)}$ is the vector of all parameters except v .

Observe that we should simulate samples for β_{pj} , $j = 0, 1, \dots, L$ considering the Metropolis-Hastings algorithm (Smith and Roberts, 1993) since their conditional distributions are unknown.

Starting with initial values $\theta^{(0)}$, we simulate samples of the joint posterior distribution (2.7) following the steps:

- (i) generate a sample of D_{i1}^* , $i = 1, \dots, n$, from the conditional Bernoulli distribution with success probability (2.3) given $T_i = 1$;

(ii) generate a sample of D_{i2}^* , $i = 1, \dots, n$, from the conditional Bernoulli distribution with

success probability (2.4) given $T_i = 0$;

(iii) generate a sample from the conditional distributions $\Pi(\theta_1 | \theta_{(\theta_1)}, \mathbf{t}, \mathbf{d}^*)$,
 $\dots, \Pi(\theta_L | \theta_{(\theta_L)}, \mathbf{t}, \mathbf{d}^*)$.

A special case is given when we do not have the presence of covariates (Joseph *et al.*, 1995). In this case, we assume Beta prior distributions for S_E and S_P given in (2.2) and a Beta distribution for p with hyperparameters u and v considering u and v known. Also assuming the introduction of the latent variables D_{i1}^* given $T_i = 1$ and D_{i2}^* given $T_i = 0$, the joint posterior distribution for $\theta' = (S_E, S_P, p)$ is given by

$$\begin{aligned} \Pi(\theta, \mathbf{d}^* | \mathbf{t}) \propto & S_E^{a + \sum_{i=1}^n t_i d_{i1}^* - 1} (1 - S_E)^{b + \sum_{i=1}^n (1 - t_i) d_{i2}^* - 1} \times \\ & \times S_P^{c + \sum_{i=1}^n (1 - t_i) (1 - d_{i2}^*) - 1} (1 - S_P)^{d + \sum_{i=1}^n t_i (1 - d_{i1}^*) - 1} \times \\ & \times p_i^{u + \sum_{i=1}^n t_i d_{i1}^* + \sum_{i=1}^n (1 - t_i) d_{i2}^* - 1} \times \\ & \times (1 - p_i)^{v + \sum_{i=1}^n t_i (1 - d_{i1}^*) + \sum_{i=1}^n (1 - t_i) (1 - d_{i2}^*) - 1}. \end{aligned} \quad (2.9)$$

The conditional posterior distributions for the Gibbs sampling algorithm are then given by

$$\begin{aligned} \text{(a) } S_E | \theta_{(S_E)}, \mathbf{t}, \mathbf{d}^* & \sim \text{Beta} \left(a + \sum_{i=1}^n d_{i1}^* t_i, b + \sum_{i=1}^n d_{i2}^* (1 - t_i) \right), \\ \text{(b) } S_P | \theta_{(S_P)}, \mathbf{t}, \mathbf{d}^* & \sim \text{Beta} \left(c + \sum_{i=1}^n (1 - d_{i2}^*) (1 - t_i), d + \sum_{i=1}^n t_i (1 - d_{i1}^*) \right), \\ \text{(c) } p | \theta_{(p)}, \mathbf{t}, \mathbf{d}^* & \sim \text{Beta}(u', v'), \end{aligned} \quad (2.10)$$

where

$$u' = u + \sum_{i=1}^n d_{i1}^* t_i + \sum_{i=1}^n d_{i2}^* (1 - t_i) \quad \text{and} \quad v' = v + \sum_{i=1}^n t_i (1 - d_{i1}^*) + \sum_{i=1}^n (1 - t_i) (1 - d_{i2}^*).$$

We also can consider here the performance measures estimation of $\omega \geq 2$ independent diagnostic tests applied to a group of individuals, where none of these tests can be considered the gold standard. Letting S_{E_w} be the sensitivity of the test w and letting S_{P_w} be the specificity of the test w , we rewrite the joint distribution for D_{i1}^* , $i = 1, \dots, n$, as

$$f(d_{11}^*, \dots, d_{n1}^*) \propto \prod_{i=1}^n \prod_{w=1}^{\omega} \frac{(p_i S_{E_w})^{t_{w_i} d_{i1}^*} [(1 - p_i) (1 - S_{P_w})]^{t_{w_i} (1 - d_{i1}^*)}}{[p_i S_{E_w} + (1 - p_i) (1 - S_{P_w})]^{t_{w_i}}}, \quad (2.11)$$

and the joint distribution for D_{i2}^* , $i = 1, \dots, n$, as

$$f(d_{12}^*, \dots, d_{n2}^*) \propto \prod_{i=1}^n \prod_{w=1}^{\omega} \frac{[p_i (1 - S_{E_w})]^{(1-t_{w_i})d_{i2}^*} [(1-p_i) S_{P_w}]^{(1-t_{w_i})(1-d_{i2}^*)}}{[p_i (1 - S_{E_w}) + (1-p_i) S_{P_w}]^{1-t_{w_i}}}, \quad (2.12)$$

where t_{w_i} is an observation of the random variable T_{w_i} related to result of the test w . Let us assume independent Beta prior distributions for S_{E_w} and S_{P_w} , $w = 1, \dots, \omega$. The subsequent development to obtain the conditional distributions for the Gibbs sampling algorithm is analogous to that one showed above.

3 Model formulation assuming verified and unverified individuals

In medical applications, if we consider only the verified cases we could obtain biased estimators for S_E and S_P , as pointed out in Begg and McNeil (1987), Begg (1987), Begg and Greenes (1983) and Zhou (1993). This situation occurs when only part of the sampled individuals are verified about their real disease status by a procedure generically denominated in the medical literature by "gold standard" (Hui and Zhou, 1998).

Bias for the estimators of S_E and S_P are directly related to the selection for verification of the individuals to check for their real disease status. The selection of individuals for verification usually depends on their test outcome. This situation occurs, for instance, in studies of the performance of nuchal translucency measurement in the diagnosis of chromosomal abnormalities (as Down syndrome), where the gold standard is fetal karyotype (Mol *et al.*, 1999). This verification procedure is generally restricted to women with an increased fetal nuchal translucency thickness. For other diseases, usually symptoms, signs and clinical examinations influence the mechanism of selection of individuals for verification. In these cases, we could have superestimation of S_E since individuals with positive test results have a larger probability of verification and underestimation of S_P since individuals with negative test results are underrepresented.

Let V be a random variable related to the verification by a gold standard, where $V = 1$ (or \bar{V}) denotes a verified individual and $V = 0$ (or \bar{V}) denotes an unverified individual. In Table 1, we have the probabilities in the cross-tabulation of the variables V , D and T considering verified and unverified individuals by a gold standard.

Table 1 Probabilities in the cross-tabulation of the variables V , D , and T

	verified ($V = 1$)		unverified ($V = 0$)
	$D = 1$	$D = 0$	
$T = 1$	$p\lambda_{11}S_E$	$(1-p)\lambda_{01}(1-S_P)$	$p(1-\lambda_{11})S_E + (1-p)(1-\lambda_{01})(1-S_P)$
$T = 0$	$p\lambda_{10}(1-S_E)$	$(1-p)\lambda_{00}S_P$	$p(1-\lambda_{10})(1-S_E) + (1-p)(1-\lambda_{00})S_P$

Let us assume that D and V are independent random variables. In Table 1, we have $\lambda_{11} = P(V|TD)$, $\lambda_{01} = P(V|T\bar{D})$, $\lambda_{10} = P(V|\bar{T}D)$, $\lambda_{00} = P(V|\bar{T}\bar{D})$, $S_E = P(T|D)$ and $S_P = P(\bar{T}|\bar{D})$. For the calculation of the probabilities of Table 1, observe that $P(T = 1, D = 1, V = 1) = P(TDV) = P(D)P(V|TD)P(T|D) = P\lambda_{11}S_E$. Also, $P(T = 1, V = 0) = P(T\bar{V}) = P(D)[1 - P(V|TD)]P(T|D) + [1 - P(D)][1 - P(V|T\bar{D})]P(\bar{T}|\bar{D}) = p(1 - \lambda_{11})S_E + (1 - p)(1 - \lambda_{01})(1 - S_P)$. In the same way, we find the other probabilities given in Table 1.

Considering the data given by $\mathcal{D}_i = (d_i, t_i, v_i)$, $i = 1, \dots, n$, where $d_i = 1$ (D) or 0 (\bar{D}); $t_i = 1$ (T) or 0 (\bar{T}), and $v_i = 1$ (V) or 0 (\bar{V}), the likelihood function for $\theta_1 = (\lambda_{11}, \lambda_{10}, \lambda_{01}, \lambda_{00}, S_E, S_P, p)$ is given by

$$\begin{aligned} L(\theta_1) &= \prod_{i=1}^n (p\lambda_{11}S_E)^{d_i t_i v_i} [p\lambda_{10}(1 - S_E)]^{d_i(1-t_i)v_i} \\ &\quad \times [(1-p)\lambda_{01}(1 - S_P)]^{(1-d_i)t_i v_i} [(1-p)\lambda_{00}S_P]^{(1-d_i)(1-t_i)v_i} \\ &\quad \times [p(1 - \lambda_{11})S_E + (1-p)(1 - \lambda_{01})(1 - S_P)]^{t_i(1-v_i)} \\ &\quad \times [p(1 - \lambda_{10})(1 - S_E) + (1-p)(1 - \lambda_{00})S_P]^{(1-t_i)(1-v_i)}. \end{aligned} \quad (3.1)$$

In the presence of a vector of covariates $\mathbf{X}'_i = (X_{0i}, X_{1i}, \dots, X_{Li})$, let us assume the logit links, as in (1.2), for p_i , S_{E_i} , S_{P_i} , λ_{11_i} , λ_{01_i} , λ_{10_i} , and λ_{00_i} given by

$$v_{li} = \frac{\exp\left(\sum_{j=0}^L \beta_{lj}x_{ji}\right)}{1 + \exp\left(\sum_{j=0}^L \beta_{lj}x_{ji}\right)}, \quad (3.2)$$

for $l = 1, 2, \dots, 7$; $X_{0i} = 1$; $v_{1_i} = p_i$; $v_{2_i} = S_{E_i}$; $v_{3_i} = S_{P_i}$; $v_{4_i} = \lambda_{11_i}$; $v_{5_i} = \lambda_{01_i}$; $v_{6_i} = \lambda_{10_i}$ and $v_{7_i} = \lambda_{00_i}$, $i = 1, \dots, n$. In this way, we have a vector of parameters given by $\theta'_2 = (\beta_1, \beta_2, \dots, \beta_7)$, where $\beta'_1 = (\beta_{10}, \beta_{11}, \dots, \beta_{1L})$, $\beta'_2 = (\beta_{20}, \beta_{21}, \dots, \beta_{2L}), \dots, \beta'_7 = (\beta_{70}, \beta_{71}, \dots, \beta_{7L})$.

4 A Bayesian analysis in the presence of covariates

Let us consider the model given in Table 1 assuming the verified and the unverified cases and the presence of a vector of covariates \mathbf{X}'_i , $i = 1, \dots, n$, associated to each individual. Assuming prior independence among the parameters, consider the following prior densities for β_{lj} :

$$\beta_{lj} \sim N(a_{lj}, b_{lj}^2); \quad a_{lj}, b_{lj} \text{ known}, \quad (4.1)$$

where $l = 1, 2, \dots, 7$; $j = 0, 1, \dots, L$. We also assume the introduction of latent variables $\mathbf{D}^*_i = (D^*_{i1}, D^*_{i2})$ (see Section 2) where D^*_{i1} given $V_i = 0$ and $T_i = 1$ and D^*_{i2} given $V_i = 0$ and $T_i = 0$, $i = 1, \dots, n$.

That is,

(a) D_{i1}^* given $V_i = 0$ and $T_i = 1$ is a random variable with a Bernoulli distribution with success probability given by

$$h_{1i} = P(D_{i1}^* | \bar{V}_i, T_i) = \frac{p_i (1 - \lambda_{11_i}) S_{E_i}}{p_i (1 - \lambda_{11_i}) S_{E_i} + (1 - p_i) (1 - \lambda_{01_i}) (1 - S_{P_i})}. \quad (4.2)$$

The joint distribution for D_{i1}^* , $i = 1, \dots, n$ is given by

$$\begin{aligned} f(d_{11}^*, \dots, d_{n1}^*) &\propto \prod_{i=1}^n \frac{[p_i (1 - \lambda_{11_i}) S_{E_i}]^{t_i (1 - v_i) d_{i1}^*}}{[p_i (1 - \lambda_{11_i}) S_{E_i} + (1 - p_i) (1 - \lambda_{01_i}) (1 - S_{P_i})]^{(1 - v_i) t_i}} \\ &\times \prod_{i=1}^n \frac{[(1 - p_i) (1 - \lambda_{01_i}) (1 - S_{P_i})]^{t_i (1 - v_i) (1 - d_{i1}^*)}}{[p_i (1 - \lambda_{11_i}) S_{E_i} + (1 - p_i) (1 - \lambda_{01_i}) (1 - S_{P_i})]^{(1 - v_i) t_i}}. \end{aligned} \quad (4.3)$$

(b) D_{i2}^* given $V_i = 0$ and $T_i = 0$ is a random variable with a Bernoulli distribution with success probability given by

$$h_{2i} = P(D_{i2}^* | \bar{V}_i, \bar{T}_i) = \frac{p_i (1 - \lambda_{10_i}) (1 - S_{E_i})}{p_i (1 - \lambda_{10_i}) (1 - S_{E_i}) + (1 - p_i) (1 - \lambda_{00_i}) S_{P_i}}. \quad (4.4)$$

The joint distribution for D_{i2}^* , $i = 1, \dots, n$ is given by

$$\begin{aligned} f(d_{12}^*, \dots, d_{n2}^*) &\propto \prod_{i=1}^n \frac{[p_i (1 - \lambda_{10_i}) S_{E_i}]^{(1 - t_i) (1 - v_i) d_{i2}^*}}{[p_i (1 - \lambda_{10_i}) (1 - S_{E_i}) + (1 - p_i) (1 - \lambda_{00_i}) S_{P_i}]^{(1 - v_i) (1 - t_i)}} \\ &\times \prod_{i=1}^n \frac{[(1 - p_i) (1 - \lambda_{00_i}) (1 - S_{P_i})]^{(1 - t_i) (1 - v_i) (1 - d_{i2}^*)}}{[p_i (1 - \lambda_{10_i}) (1 - S_{E_i}) + (1 - p_i) (1 - \lambda_{00_i}) S_{P_i}]^{(1 - v_i) (1 - t_i)}}. \end{aligned} \quad (4.5)$$

Combining (4.3) and (4.5) with the prior distribution for θ_2' and the likelihood function, we have the joint posterior distribution for $\theta_2' = (\beta_1, \beta_2, \dots, \beta_7)$ and \mathbf{d}^*

given by

$$\begin{aligned}
\Pi(\theta_2, \mathbf{d}^* | \mathcal{D}, \mathbf{t}) &\propto \Pi(\theta_2) \left[\prod_{i=1}^n \lambda_{00i}^{r_i^{(001)}} (1 - \lambda_{00i})^{s_{2i}^{(000)}} \right] \left[\prod_{i=1}^n \lambda_{10i}^{r_i^{(101)}} (1 - \lambda_{10i})^{s_{2i}^{(100)}} \right] \\
&\times \left[\prod_{i=1}^n \lambda_{01i}^{r_i^{(011)}} (1 - \lambda_{01i})^{s_{1i}^{(010)}} \right] \left[\prod_{i=1}^n \lambda_{11i}^{r_i^{(111)}} (1 - \lambda_{11i})^{s_{1i}^{(110)}} \right] \\
&\times \left[\prod_{i=1}^n S_{P_i}^{r_i^{(001)} + s_{2i}^{(000)}} (1 - S_{P_i})^{r_i^{(011)} + s_{1i}^{(010)}} \right] \\
&\times \left[\prod_{i=1}^n S_{E_i}^{r_i^{(111)} + s_{1i}^{(110)}} (1 - S_{E_i})^{r_i^{(101)} + s_{2i}^{(100)}} \right] \\
&\times \left[\prod_{i=1}^n p_i^{r_i^{(111)} + r_i^{(101)} + s_{1i}^{(110)} + s_{2i}^{(100)}} (1 - p_i)^{r_i^{(011)} + r_i^{(001)} + s_{1i}^{(010)} + s_{2i}^{(000)}} \right],
\end{aligned} \tag{4.6}$$

where $\Pi(\theta_2)$ is the joint prior distribution for θ_2 (see (4.1)), $r_i^{(111)} = d_i t_i v_i$, $r_i^{(011)} = (1 - d_i) t_i v_i$, $r_i^{(101)} = d_i (1 - t_i) v_i$, $r_i^{(001)} = (1 - d_i) (1 - t_i) v_i$, $s_{1i}^{(110)} = d_{i1}^* t_i (1 - v_i)$, $s_{1i}^{(010)} = (1 - d_{i2}^*) t_i (1 - v_i)$, $s_{2i}^{(100)} = d_{i2}^* (1 - t_i) (1 - v_i)$, $s_{2i}^{(000)} = (1 - d_{i2}^*) (1 - t_i) (1 - v_i)$, $i = 1, \dots, n$.

We simulate samples from the joint distribution for $\theta'_2 = (\beta_1, \beta_2, \dots, \beta_7)$ using the Metropolis-Hastings algorithm.

In the situation where we do not have the presence of covariates, we have a vector of parameters given by $\theta'_1 = (v_1, v_2, \dots, v_7)$, where $v_1 = p$, $v_2 = S_E$, $v_3 = S_P$, $v_4 = \lambda_{11}$, $v_5 = \lambda_{01}$, $v_6 = \lambda_{10}$ and $v_7 = \lambda_{00}$. Assuming prior independence among the parameters, let us consider $Beta(a_{1j}, b_{1j})$ prior distributions for v_j , $j = 1, \dots, 7$, with a_{1j} and b_{1j} known hyperparameters. Also considering the introduction of the latent variables D_{ik}^* , $i = 1, \dots, n$, the joint posterior distribution for θ_1 and \mathbf{d}^* is given by

$$\begin{aligned}
\Pi(\theta_1, \mathbf{d}^* | \mathcal{D}, \mathbf{t}) &\propto p^{a_{11} + G_1 - 1} (1 - p)^{b_{11} + G_2 - 1} S_E^{a_{12} + G_3 - 1} (1 - S_E)^{b_{12} + G_4 - 1} \\
&\times S_P^{a_{13} + G_5 - 1} (1 - S_P)^{b_{13} + G_6 - 1} \lambda_{01}^{a_{14} + G_7 - 1} (1 - \lambda_{01})^{b_{14} + G_8 - 1} \\
&\times \lambda_{10}^{a_{15} + G_9 - 1} (1 - \lambda_{10})^{b_{15} + G_{10} - 1} \lambda_{00}^{a_{16} + G_{11} - 1} (1 - \lambda_{00})^{b_{16} + G_{12} - 1} \\
&\times \lambda_{11}^{a_{17} + G_{13} - 1} (1 - \lambda_{11})^{b_{17} + G_{14} - 1},
\end{aligned} \tag{4.7}$$

where

$$\begin{aligned}
G_1 &= \sum_{i=1}^n d_i v_i + \sum_{j=1}^2 \sum_{i=1}^n (1 - v_i) d_{ij}^*, \\
G_2 &= \sum_{i=1}^n (1 - d_i) v_i + \sum_{j=1}^2 \sum_{i=1}^n (1 - v_i) (1 - d_{ij}^*), \\
G_3 &= \sum_{i=1}^n d_i t_i v_i + \sum_{i=1}^n t_i (1 - v_i) d_{i1}^*, \\
G_4 &= \sum_{i=1}^n d_i (1 - t_i) + \sum_{i=1}^n (1 - t_i) (1 - v_i) d_{i2}^*, \\
G_5 &= \sum_{i=1}^n (1 - d_i) (1 - t_i) v_i + \sum_{i=1}^n (1 - t_i) (1 - v_i) (1 - d_{i2}^*), \\
G_6 &= \sum_{i=1}^n (1 - d_i) t_i v_i + \sum_{i=1}^n t_i (1 - v_i) (1 - d_{i1}^*), \\
G_7 &= \sum_{i=1}^n (1 - d_i) t_i v_i, \\
G_8 &= \sum_{i=1}^n t_i (1 - v_i) (1 - d_{i1}^*), \\
G_9 &= \sum_{i=1}^n d_i (1 - t_i) v_i, \\
G_{10} &= \sum_{i=1}^n (1 - t_i) (1 - v_i) d_{i2}^*, \\
G_{11} &= \sum_{i=1}^n (1 - d_i) (1 - t_i) v_i, \\
G_{12} &= \sum_{i=1}^n (1 - t_i) (1 - v_i) (1 - d_{i2}^*), \\
G_{13} &= \sum_{i=1}^n d_i t_i v_i, \text{ and} \\
G_{14} &= \sum_{i=1}^n t_i (1 - v_i) d_{i1}^*.
\end{aligned}$$

The conditional distributions for the Gibbs sampling algorithm are given by

$$\begin{aligned}
p|\theta_{(p)}, \mathcal{D}, \mathbf{d}^* &\sim \text{Beta}(a_{11} + G_1, b_{11} + G_2), \\
S_E|\theta_{(S_E)}, \mathcal{D}, \mathbf{d}^* &\sim \text{Beta}(a_{12} + G_3, b_{12} + G_4), \\
S_P|\theta_{(S_P)}, \mathcal{D}, \mathbf{d}^* &\sim \text{Beta}(a_{13} + G_5, b_{13} + G_6), \\
\lambda_{01}|\theta_{(\lambda_{01})}, \mathcal{D}, \mathbf{d}^* &\sim \text{Beta}(a_{14} + G_7, b_{14} + G_8), \\
\lambda_{10}|\theta_{(\lambda_{10})}, \mathcal{D}, \mathbf{d}^* &\sim \text{Beta}(a_{15} + G_9, b_{15} + G_{10}), \\
\lambda_{00}|\theta_{(\lambda_{00})}, \mathcal{D}, \mathbf{d}^* &\sim \text{Beta}(a_{16} + G_{11}, b_{16} + G_{12}), \\
\lambda_{11}|\theta_{(\lambda_{11})}, \mathcal{D}, \mathbf{d}^* &\sim \text{Beta}(a_{17} + G_{13}, b_{17} + G_{14}),
\end{aligned} \tag{4.8}$$

where, for instance, $\theta_{(p)}$ is the vector θ_1 without the parameter p .

5 Applications to real data sets

In this section we apply the methods presented in the previous sections in three real data sets. We used the software SAS (proc IML) to perform all the simulations. The convergence of the Gibbs samples was monitored by standard existing methods (Geweke, 1992) available in CODA package (Best *et al.*, 1995).

5.1 Example 1: a diagnostic test for coronary artery disease

To illustrate the applicability of the proposed model, let us firstly consider an example with one diagnostic test and three covariates. Kosinski and Barnhart (2003) introduced a data set from the area of cardiology, where the single-photon-emission computed tomography (SPECT) stress thallium is considered as a diagnostic test for coronary artery disease. The authors consider 2,688 patients without known history of myocardial infarction and with no prior coronary angiography. Let X_1 be a binary variable related to gender (1 = male, 0 = female), X_2 is a variable related to stress mode (1 = induced by intravenous dipyridamole infusion, 0 = induced by exercise), and X_3 is related to age (1 = age ≥ 60 , 0 = otherwise, dichotomized in this manner by clinical reasons). Since we are not considering a gold standard, we used the proposed methodology to estimate the sensitivity and specificity measures of SPECT. From the respective conditional posterior distributions, a chain of 40,000 iterations was run for each parameter after preliminary burn-in of 1,000 iterations. For each parameter we considered every 10th draw. Prior distributions for β_{10} , β_{20} and β_{30} were assumed with hyperparameters based in estimates obtained in a preliminary analysis without covariates. All the other hyperparameter values were chosen to be noninformative.

Table 2 shows the posterior summaries for each parameter and its interpretation in the model. We note that the 95% credible intervals for β_{11} , β_{12} and β_{13} included the value 0, suggesting no effect of the covariates on S_E . Analogously, we also note a suggestion of no effect of the covariates on p . However, Table 2 shows significative effects of all covariates on S_P . From the simulated values for

the parameters, we have, for example, an estimate of 0.641 for S_P when $X_1 = 0$, $X_2 = 0$ and $X_3 = 0$, and an estimate of 0.482 for S_P when $X_1 = 0$, $X_2 = 1$ and $X_3 = 0$.

Table 2 *Posterior summaries (S.D.: standard deviation)*

parameter	measure	mean	SD	95% credible interval	
β_{10}	intercept for S_E	1.445	0.1708	1.208	1.772
β_{20}	intercept for S_P	0.578	0.0205	0.525	0.599
β_{30}	intercept for p	-2.816	0.1617	-2.995	-2.403
β_{11}	effect of X_1 on S_E	-0.024	0.5878	-0.950	0.956
β_{21}	effect of X_1 on S_P	-0.604	0.0852	-0.762	-0.423
β_{31}	effect of X_1 on p	0.031	0.5746	-0.943	0.953
β_{12}	effect of X_2 on S_E	-0.033	0.5836	-0.960	0.950
β_{22}	effect of X_2 on S_P	-0.646	0.1024	-0.835	-0.436
β_{32}	effect of X_2 on p	0.025	0.593	-0.949	0.962
β_{13}	effect of X_3 on S_E	0.083	0.5939	-0.952	0.965
β_{23}	effect of X_3 on S_P	-0.246	0.1066	-0.425	-0.008
β_{33}	effect of X_3 on p	-0.193	0.5621	-0.969	0.919

5.2 Example 2: screening of a drug in the urine of leprosy patients

In Table 3, we have the test results for the presence in urine of a prescribed drug, Dapsone (DDS), for 277 patients suffering from leprosy (data set introduced by Nagelkerke *et al.*, 1988). Three different biochemical tests were used for screening: the ELISA test, the Haemagglutination test and the Spot test. However, none of these tests are infallible and the diagnosis of Dapsone in urine is limited by the lack of a gold standard. Table 2 displays how the test results were distributed among the study participants. Table 3 also shows the frequency distribution of test outcomes by level of education.

Table 3 *Frequency distribution of three diagnostic tests findings by level of education*

biochemical tests			total	primary schooling		
ELISA test	Haemagglutination test	Spot test		none	incomplete	complete
positive	positive	positive	122	18	52	52
positive	positive	negative	13	3	6	4
positive	negative	positive	14	3	8	3
positive	negative	negative	17	2	7	8
negative	positive	positive	8	2	4	2
negative	positive	negative	10	2	5	3
negative	negative	positive	10	4	6	0
negative	negative	negative	83	15	44	24

Using the proposed methodology, we estimated the sensitivity and specificity measures of ELISA, Haemagglutination and Spot tests, without a reference test (gold standard). In this model, X is a covariate related to primary schooling, coded as "none", "incomplete" or "complete". To introduce this variable in the model, we use a set of dummy variables X_1 and X_2 , where X_1 and X_2 are both equal to zero when a patient never went to primary school; X_1 is equal to 1 and X_2 is equal to 0 when a patient has incomplete primary schooling; and when a patient has complete primary schooling, we used $X_1 = 0$ and $X_2 = 1$. From the respective conditional posterior distributions we generated a chain of 100,000 iterations, and in order to diminish some effect of the initial parameters values, we discarded the first 5,000 elements of each chain. For each parameter we considered every 10^{th} draw. The results for a Bayesian analysis without the covariates are provided in Table 4. Bayesian inference was carried out using vague prior distributions for all model parameters. The hyperparameter values for the prior distributions are 0.5. A small sensitivity analysis was made by choosing other hyperparameter values, but their choice do not modify substantially the results presented below and the correspondent results are omitted here.

Table 4 *Posterior summaries for the prevalence of presence of Dap-
sone in urine (p), sensitivities (S_{E_1}, S_{E_2} and S_{E_3}) and
specificities (S_{P_1}, S_{P_2} and S_{P_3}) of ELISA, Haemagglutina-
tion and Spot tests (S.D.: standard deviation)*

biochemical test	parameter	posterior summaries		95% credible interval	
		mean	SD		
ELISA test	S_{E_1}	0.943	0.024	0.891	0.984
	S_{P_1}	0.833	0.040	0.748	0.906
Haemagglutination test	S_{E_2}	0.907	0.029	0.845	0.958
	S_{P_2}	0.891	0.034	0.814	0.952
Spot test	S_{E_3}	0.914	0.028	0.853	0.965
	S_{P_3}	0.892	0.035	0.814	0.953
	p	0.556	0.034	0.487	0.622

Table 5 shows the posterior summaries for the parameters in the Bayesian analysis of the model that considers the level of education as a covariate. The parameters β_{11} to β_{51} are related to category "incomplete primary schooling" versus "none", and the parameters β_{12} to β_{52} are associated to category "complete primary schooling" versus "none". The specification of the hyperparameters was based in the results of three preliminary models without covariates, each one for a distinct level of education. We are in this manner adopting an empirical Bayesian approach (Carlin and Louis, 2000), where the prior distributions are coming of a preliminary analysis of the data. We note that the 95% credible intervals for β_{11} to β_{71} and β_{12} to β_{72} included the value 0, suggesting no effect of the level of education on p and the performance measures.

Table 5 *Posterior summaries, model with covariate (S.D.: standard deviation)*

parameter	measure	mean	SD	95% credible interval	
β_{10}	intercept for S_{E_1}	2.8584	0.3731	2.1533	3.6244
β_{20}	intercept for S_{E_2}	2.3132	0.3374	1.6847	2.9929
β_{30}	intercept for S_{E_3}	2.3853	0.3319	1.7321	3.0688
β_{40}	intercept for S_{P_1}	1.6728	0.2914	1.1260	2.2671
β_{50}	intercept for S_{P_1}	2.1553	0.3306	1.5017	2.8043
β_{60}	intercept for S_{P_1}	2.0996	0.3391	1.4397	2.7861
β_{70}	intercept for p	0.1487	0.2205	-0.2837	0.5829
β_{11}	effect of X_1 on S_{E_1}	-0.0099	0.4194	-0.7962	0.8260
β_{21}	effect of X_1 on S_{E_2}	-0.1364	0.3884	-0.8957	0.6520
β_{31}	effect of X_1 on S_{E_3}	0.0105	0.3886	-0.7594	0.7841
β_{41}	effect of X_1 on S_{P_1}	0.1272	0.3651	-0.5879	0.8401
β_{51}	effect of X_1 on S_{P_2}	0.0590	0.3871	-0.6559	0.8087
β_{61}	effect of X_1 on S_{P_3}	-0.0224	0.3891	-0.7716	0.7580
β_{71}	effect of X_1 on p	-0.0624	0.2573	-0.5571	0.4316
β_{12}	effect of X_2 on S_{E_1}	0.2167	0.4243	-0.6168	1.0647
β_{22}	effect of X_2 on S_{E_2}	0.3175	0.3964	-0.4133	1.1261
β_{32}	effect of X_2 on S_{E_3}	0.1684	0.3904	-0.5656	0.9363
β_{42}	effect of X_2 on S_{P_1}	-0.3120	0.3691	-1.0145	0.4163
β_{52}	effect of X_2 on S_{P_2}	0.0252	0.4175	-0.7600	0.8471
β_{62}	effect of X_2 on S_{P_3}	0.4954	0.4215	-0.3159	1.3489
β_{72}	effect of X_2 on p	0.3187	0.2699	-0.2063	0.8509

Table 6 summarizes the posterior estimates of S_{E_1} , S_{E_2} , S_{E_3} , S_{P_1} , S_{P_2} , S_{P_3} and p by level of education, calculated from the simulated values related to the parameters in θ_2 . These results suggest again that the prevalence of presence of Dapsone in urine and the performance measures of the tests are not related to the level of education.

5.3 Example 3: Hybrid capture II in the diagnosis of cervical intraepithelial neoplasia

In statistical analysis of the performance of usual and new techniques used in the detection of cervical lesions usually a gold standard is unavailable. By ethical reasons, a portion of the sampled women may not be verified by a usual gold standard as the cervical biopsy, generally an invasive procedure, not usually applied to asymptomatic women. To evaluate the performance of hybrid capture II (HC-II) for DNA-HPV in the diagnosis of high-grade cervical intraepithelial neoplasia (CIN), Santos *et al.* (2003) consider a data set on 105 women attended at a reference service from August 2000 to June 2001. Hybrid capture II was conducted in all the patients, but cervical biopsies (the gold standard) were taken only in 91 women. The results of the study are showed in Table 7.

Table 6 *Posterior summaries for the prevalence of presence of Dap-
sone in urine (p), sensitivities (S_{E_1}, S_{E_2} and S_{E_3}) and
specificities (S_{P_1}, S_{P_2} and S_{P_3}) of ELISA, Haemagglutina-
tion and Spot tests, by level of education (S.D.: standard
deviation; 95% CI: 95% credible interval)*

biochemical test	parameter	Primary schooling								
		none			incomplete			complete		
		mean	95% CI		mean	95% CI		mean	95% CI	
ELISA test	S_{E_1}	0.943	0.896	0.974	0.940	0.884	0.979	0.952	0.898	0.983
	S_{P_1}	0.838	0.755	0.906	0.853	0.760	0.926	0.790	0.662	0.893
Haemagglutination test	S_{E_2}	0.906	0.844	0.952	0.892	0.807	0.953	0.928	0.864	0.972
	S_{P_2}	0.892	0.817	0.943	0.896	0.814	0.955	0.891	0.791	0.957
Spot test	S_{E_3}	0.912	0.849	0.956	0.911	0.841	0.963	0.923	0.855	0.968
	S_{P_3}	0.887	0.808	0.942	0.883	0.791	0.948	0.925	0.850	0.971
	p	0.537	0.429	0.642	0.521	0.435	0.610	0.614	0.515	0.707

Table 7 *Distribution of DNA-HPV HC-II testing results by cervical lesion as ascertained by cervical biopsy*

DNA-HPV HC-II	cervical biopsy ($V = 1$)		unverified ($V = 0$)	total
	$D = 1$	$D = 0$		
$T = 1$	17	36	3	56
$T = 0$	3	35	11	49
total	20	71	14	105

First, we conducted a Bayesian analysis without covariate information. Considering the likelihood function for θ_1 given by (3.1), we assume the Beta prior distributions for all parameters. Prior distributions for S_E and E_S were assumed as proposed in Section 2 with hyperparameters $a = 7.2$, $b = 1.6$, $c = 2.8$, $d = 2.4$. All the other hyperparameter values are equal to 0.5, that is, these prior distributions were chosen to be noninformative. The values of the hyperparameters a , b , c e d were based in classical maximum likelihood estimates (see Begg and Greenes, 1983) obtained in a preliminary analysis. We used thus an empirical Bayesian modelling (see, for example, Carlin and Louis, 2000). From the respective conditional posterior distributions we generated a chain of 100,000 iterations. In Table 8, we have the posterior summaries obtained for each parameter, where the 5,000 first iterations of each parameter were discarded (the "burn-in samples"). For each parameter we considered every 20th draw. A small sensitivity analysis was made by choosing other hyperparameter values (for example, considering all hyperparameters equal to 0.5; in this way the prior distributions are noninformative with respect to the data), but their choice do not modify substantially the results presented below and the correspondent results are omitted here. The results in Table 1 indicate that the posterior probability of an individual to be verified, given a positive test, is very large. The probabilities of verification $\lambda_{11} = P(V|TD)$ and $\lambda_{01} = P(V|T\bar{D})$ are estimated in 0.969 and 0.914, respectively. We also noted in Table 8 a very large 95% credible interval for λ_{10} . This can be due to an unsatisfactory choice of the hyperparameters values for λ_{11} , λ_{01} , λ_{10} and λ_{00} . In this example, we assumed that the probability of verification is a priori the same for both positive and negative results, and this may be unrealistic in practice. We believe that more accurate results would be given if we incorporated prior opinion of clinical experts about these probabilities of verification.

In a second instance, we introduced in the model the age of the women as a covariate. From the conditional densities for θ_2 we generated 100,000 Gibbs samples. From this chain, we discarded the first 5,000 samples. For each parameter, we considered every 50th draw. The convergence was observed for all parameters. Under an empirical Bayes framework, the values of the hyperparameters β_{10} to β_{70} were based in the posterior summaries showed in Table 8. The values of the hyperparameters a_{l1} , $l = 1, \dots, 7$, were set to be zero, and the variances b_{l1} , $l = 1, \dots, 7$, were set to be relatively large, or say, around 10. With very large values for b_{l1} we could have problems with the convergence. Table 9 shows the posterior summaries for the parameters in the Bayesian analysis of the model. The 95% credible intervals for β_{11} to β_{71} included the value 0, suggesting no effect

of dichotomized age on any measure. In Table 10 we show the posterior summaries for S_E , S_P , p , λ_{11} , λ_{01} , λ_{10} and λ_{00} on the basis of the values generated for the parameters in θ_2 , by age groups. It is visible in Table 10 that the Bayesian estimates of S_E , S_P , p , λ_{11} , λ_{01} , λ_{10} and λ_{00} are not influenced by age.

Table 8 *Posterior summaries (S.D.: standard deviation)*

parameter	prior distribution	posterior summaries			
		mean	SD	95% credible interval	
S_E	$Beta(7.2, 1.6)$	0.778	0.1011	0.554	0.933
S_P	$Beta(2.8, 2.4)$	0.526	0.0570	0.413	0.637
p	$Beta(0.5, 0.5)$	0.218	0.0505	0.135	0.331
λ_{11}	$Beta(0.5, 0.5)$	0.969	0.0432	0.849	0.998
λ_{01}	$Beta(0.5, 0.5)$	0.914	0.0436	0.812	0.979
λ_{10}	$Beta(0.5, 0.5)$	0.632	0.2794	0.123	0.998
λ_{00}	$Beta(0.5, 0.5)$	0.805	0.0885	0.641	0.993

Table 9 *Posterior summaries, model with covariate (S.D.: standard deviation)*

parameter	measure	posterior summaries			
		mean	SD	95% credible interval	
β_{10}	intercept for S_E	1.24730	0.28238	0.69581	1.80687
β_{20}	intercept for S_P	-0.02549	0.20070	-0.41931	0.36583
β_{30}	intercept for p	-1.21109	0.21406	-1.63256	-0.80222
β_{40}	intercept for λ_{11}	2.83752	0.30930	2.23028	3.44670
β_{50}	intercept for λ_{01}	2.39248	0.28312	1.84293	2.95666
β_{60}	intercept for λ_{10}	0.31608	0.30705	-0.28209	0.92592
β_{70}	intercept for λ_{00}	1.37572	0.26393	0.86046	1.90992
β_{11}	effect of age on S_E	-0.03306	0.30752	-0.64126	0.57247
β_{21}	effect of age on S_P	0.44212	0.24904	-0.052668	0.94119
β_{31}	effect of age on p	-0.20324	0.25685	-0.70498	0.29309
β_{41}	effect of age on λ_{11}	0.03526	0.31366	-0.57673	0.65932
β_{51}	effect of age on λ_{01}	0.10511	0.30562	-0.49306	0.72538
β_{61}	effect of age on λ_{10}	-0.02745	0.31290	-0.63822	0.58823
β_{71}	effect of age on λ_{00}	0.04391	0.27499	-0.48646	0.58519

6 Concluding remarks

In practice it is common we find situations where we have verified and unverified individuals about the real disease status after a medical diagnostic test. Considering classical procedures to obtain inference for the two probabilities of great

medical interest, the sensitivity and the specificity, we could have great difficulties. The use of MCMC methods to obtain Bayesian inference for binary data with misclassifications in the presence of covariates is a suitable way to get the posterior summaries of interest. It is important to point out that the use of MCMC methods does not require great computational expertise. These results can be of great interest in medical studies.

Table 10 *Posterior summaries for S_E , S_P , p , λ_{11} , λ_{01} , λ_{10} and λ_{00} , by age group (S.D.: standard deviation)*

parameter	posterior summaries		95% credible interval	
	mean	SD		
≤ 30 years				
S_E	0.773	0.0489	0.667	0.859
S_P	0.494	0.0497	0.396	0.590
p	0.232	0.0379	0.163	0.309
λ_{11}	0.942	0.0169	0.902	0.969
λ_{01}	0.914	0.0223	0.863	0.951
λ_{10}	0.577	0.0733	0.429	0.716
λ_{00}	0.795	0.0426	0.702	0.871
> 30 years				
S_E	0.764	0.0714	0.606	0.882
S_P	0.601	0.0582	0.483	0.710
p	0.199	0.0432	0.123	0.292
λ_{11}	0.942	0.0241	0.883	0.976
λ_{01}	0.919	0.0299	0.848	0.964
λ_{10}	0.569	0.1025	0.363	0.758
λ_{00}	0.800	0.0513	0.691	0.891

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Jorge Alberto Achcar and Francisco Louzada-Neto

Departamento de Estatística
Universidade Federal de São Carlos
CP 676 - 13565-905 - São Carlos - SP, Brasil.
E-mails: jachcar@power.ufscar.br and dfn@power.ufscar.br

Edson Zangiacomi Martinez

Departamento de Medicina Social
Faculdade de Medicina de Ribeirão Preto
Universidade de São Paulo
14049-900 - Ribeirão Preto - SP, Brasil.
E-mail: edson.martinez@ig.com.br