Brazilian Journal of Probability and Statistics (2006), 20, pp. 179–190.

A Bayesian inference for the extended skew-normal measurement error model

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Abstract: In this paper, we introduce the multivariate skew-normal model in the context of measurement error models in order to avoid data transformations or usual constraints on the parametric space. This distribution was recently discussed by Capitanio, Azzalini and Stanghellini (2003) using graphical models. The motivation is to use the conditioning argument on the unobserved true value of the explanatory variable in the two-variable measurement error model in order to get the skewness parameters as a function the original parameters of the proposed measurement error models. Inferential problems are considered from the Bayesian point of view assuming proper noninformative priors via Winbugs. The usefulness of the proposed model with errors in variables is investigated with a simulation study and real data analysis. The main advantage of the Bayesian approach is the possibility to measure the degree of belief that the true value of the explanatory variable is greater than its mean value. This constraint implies a strong asymmetry on the distribution of the unobserved true value. We end the paper by concluding that the extended skew-normal measurement error model provides flexibility in terms of skewness without making any additional assumptions to eliminate the usual identifiable problems in the measurement error models.

Key words: Noninformative prior, posterior distribution, structural error model, Winbugs.

1 Introduction

An n-dimensional random vector Z is said to have an extended multivariate random distribution with parameters $(\mu, \Omega, \lambda, \tau)$, denoted by $ESN_n(\mu, \Omega, \lambda, \tau)$, if it is continuous with density

$$\varphi_n(z, \mu, \Omega) \frac{\Phi\left(\alpha_o + \lambda^t (z - \mu)\right)}{\Phi(\tau)}, \tag{1.1}$$

where

- α_o is a function of (Ω, λ, τ) to be specified later.

- $\varphi_n(z, \mu, \Omega)$ is the n-dimensional normal density with mean μ and covariance matrix Ω.
- $\Phi(\cdot)$ is the standard N(0,1) distribution function.
- λ is an n-dimensional vector called the skewness parameter.

When $\tau = 0$, also $\alpha_o = 0$ and (1.1) reduces to the multivariate skew-normal distribution studied by Azzalini and Dalla-Valle (1996) given by

$$2\varphi(z; \mu; \Omega) \Phi\left(\lambda^t (z - \mu)\right), \tag{1.2}$$

where λ regulates the distributional shape; if $\lambda = 0$ we are back to the normal situation.

The density function (1.1) arose in Azzalini and Capitanio (1999) from a conditioning operation on (1.2); see their result (13). Also, this density was studied by Capitanio et al. (2003) and Arnold and Beaver (2000). An interesting property noticed by Azzalini and Capitanio is the closure of (1.1) under conditioning. This was possible due to the additional parameter τ which will be crucial in the development of the skew measurement error model discussed in the next section.

There are many stochastic representations for (1.1) and (1.2), however, we consider only the one introduced by Azzalini and Dalla-Valle (1996) called " conditioning method ". Arellano-Valle and Genton (2005) proposed an unified approach to obtain multivariate skew distributions which is more general than the conditioning method. See also Arellano-Valle, del Pino and San Martin (2002). Next, we formulate the conditioning method (see, Capitanio et al., 2003) as follows: Suppose that U^o is a scalar random variable and U a d-dimensional vector, such that,

$$\begin{pmatrix} U^o \\ U \end{pmatrix} \sim N_{d+1}(0; \Omega^*), \Omega^* = \begin{pmatrix} 1 & \delta^t \\ \delta & \Omega \end{pmatrix}, \tag{1.3}$$

where Ω^* is a full covariance matrix. Then, the conditional distribution of $U \mid U_o + \tau > 0$ is the $ESN_d(0, \Omega, \lambda, \tau)$, where

$$\begin{aligned} \lambda &= \frac{1}{\left(1 - \delta^t \Omega^{-1} \delta\right)^{\frac{1}{2}}} \Omega^{-1} \delta \text{ and} \\ \alpha_o &= \tau \left(1 - \delta^t \Omega^{-1} \delta\right)^{-\frac{1}{2}}. \end{aligned} \tag{1.4}$$

The main purpose of this paper is to use the above conditioning method to express the skewness parameter, λ, in terms of the original parameters involved in the measurement error models in order to avoid the usual constraints to eliminate te well-known identifiable problems (see Fuller´s book, 1987).

The paper is organized as follows: in Section 2 we formulate the normal measurement error model (Fuller, 1987) and the skew-normal measurement error model is obtained via the conditioning method already discussed in this section. In Section 3 the Bayesian analysis to estimate the parameters involved is presented. In Sections 4 and 5 a simulation study and an application to a data set analyzed in the literature without incorporating measurement errors are considered, respectively. Finally, we end the paper with a conclusion in Section 6.

2 Normal measurement error model

Regression with errors in variables, also known as the measurement error models, is fundamentally different from the simple linear regression and the problems that arise with this model are quite different. The measurement error model can be thought as a generalization of the traditional simple regression model. In this section, we consider the measurement error model relating the response variable Y_i and the auxiliary variable x_i (see details of this model in Fuller, 1987) as follows:

$$Y_i = \alpha + \beta x_i + e_i, i = 1, ..., n. \tag{2.1}$$

Assuming that x_i is the real unobserved value of the observed explanatory variable , X_i, we consider the following measurement error model

$$X_i = x_i + u_i, \tag{2.2}$$

where

$$\begin{pmatrix} e_i \\ u_i \\ x_i \end{pmatrix} \sim N_3 \left(\begin{pmatrix} 0 \\ 0 \\ \mu_x \end{pmatrix} ; diag(\sigma_e^2, \sigma_u^2, \sigma_x^2) \right)$$

are independent random vectors for $i = 1, ..., n$. The equations (2.1)-(2.2) define the structural measurement error model. When the unobserved value, x_i, is fixed this model is known in the literature as the functional model. It is well known that the above model is nonidentifiable, so, it is usual to make assumptions to solve inferential problems. Using well known properties of the multivariate normal distribution we can show that

$$(Y_i, X_i) \sim N_2 (\mu; \Omega), \tag{2.3}$$

where,

$$\mu = \begin{pmatrix} \alpha + \beta\mu_x \\ \mu_x \end{pmatrix} \text{ and } \Omega = \begin{pmatrix} \beta^2\sigma_x^2 + \sigma_e^2 & \beta\sigma_x^2 \\ \beta\sigma_x^2 & \sigma_x^2 + \sigma_u^2 \end{pmatrix}.$$

From (2.1) to (2.3) we can obtain the following results which will be very useful in the next section in order to introduce asymmetry in (2.3):

- $Cov\,(Y_i, x_i) = \beta\sigma_x^2$,
- $Cov\,(X_i, x_i) = \sigma_x^2$,
- $Cov\,(Y_i, U_i^o) = \beta\sigma_x$, where $U_i^o = \frac{x_i - \mu_x}{\sigma_x} \sim N\,(0; 1)$,
- $Cov\,(X_i, U_i^o) = \sigma_x$.

For more details of the above model from the frequentist point of view we suggest the reader to see the first chapter of Fuller´s book (1987). The purpose of this paper is to introduce asymmetry on the model (2.3) assuming the following constraint on x_i:

$$U_i^o + \tau \geq 0, \tag{2.4}$$

where the additional parameter τ is used in order to control the asymmetry of the normal distribution of x_i with respect to its mean μ_x, that is,

- $\tau = 0 \Rightarrow x_i \geq \mu_x$, in this case we have a strong asymmetry.
- $\tau = \frac{\mu_x}{\sigma_x} \Rightarrow x_i \geq 0$.
- $\tau \Rightarrow \infty$ we obtain the usual measurement error model defined by (2.2) and (2.3).

In the literature of regression model with measurement error random variables is usual to make additional assumptions like σ_u^2 is known, the variance ratio $\frac{\sigma_e^2}{\sigma_u^2}$ is known, or the reliability ratio $k_x = \frac{\sigma_x^2}{\sigma_x^2+\sigma_u^2}$ is known in order to obtain an identifiable model. In this paper, we adopt a Bayesian strategy to avoid these constraints considering the two-stage prior on τ introduced by O ´Hagan and Leornard (1976)

$$\begin{cases} \tau \sim N\left(\tau_o, \sigma^2\right), \\ \tau_o \sim N\left(0, \sigma_o^2\right), \tau_o \geq 0. \end{cases} \tag{2.5}$$

The inequality constraint on τ_o means, from (2.5), that we are worried about the right asymmetry of the real but unavailable value x_i, that is, $x_i \geq \mu_x + \sigma_x \tau$. For example, if $\sigma^2 = \sigma_o^2 = 0$, we have a strong asymmetry in x_i which means $x_i \geq \mu_x$. So, in the first stage the known variance σ^2 measures the degree of belief in the constraint $\tau_o \geq 0$ and the known variance σ_o^2 measures the degree of belief in $\tau_o = 0$. O ´Hagan and Leonard (1976) showed that the hierarchical scheme (2.5) implies a skew-normal prior for τ with skewness parameter λ^*, written $\tau \sim SN(\lambda^*)$, with density function given by

$$\varphi(\tau; \lambda^*) = 2\varphi\left\{\left(\sigma^2 + \sigma_o^2\right)^{-\frac{1}{2}} \tau\right\} \Phi\left\{\lambda^* \frac{\tau}{(\sigma^2 + \sigma_o^2)^{\frac{1}{2}}}\right\}, \tag{2.6}$$

where $\lambda^* = \frac{\sigma_o}{\sigma}$.

Here λ^* is the shape parameter which determines the prior skewness of τ. The skew-normal model (2.6) suggests us to obtain the subjective prior information about the parameters $\kappa = \sigma^2 + \sigma_o^2$ and λ^* in order to measure the belief on the constraint $x_i \geq \mu_x$ and on the positive asymmetry of τ, respectively. Consequently, assuming that κ and λ^* are known we can obtain the variances:

$$\sigma^2 = \frac{\kappa}{1 + (\lambda^*)^2} \text{ and } \sigma_o^2 = \frac{(\lambda^*)^2 \kappa}{1 + (\lambda^*)^2}. \tag{2.7}$$

Although O´Hagan and Leonard (1976) had considered the density (2.6), the first appearance of the skew-normal family of distributions dates back to Roberts (1966) and the formal definition of the family is due to Azzalini (1985). After Azzalini´s paper (1985) there have been several developments in this area (see, for example, Genton, 2004) a book with the most recent advances on the subject. More specifically, Liseo and Loperfido (2002) extended (2.5) to the multivariate case given a Bayesian interpretation for the multivariate skew-normal distribution.

3 Extended skew-normal measurement error model

In this section, we present a new measurement error model which takes into account the asymmetry of the unobserved x_i. This model is obtained via the conditioning method based on the constraint (2.4) which is the main contribution of this paper. Let U in (1.3) given by

$$U_i = \begin{pmatrix} Y_i - \alpha - \beta\mu_x \\ X_i - \mu_x \end{pmatrix} = Z_i - \mu, \text{where } \mu^{\mathrm{t}} = (\alpha + \beta\mu_{\mathrm{x}}, \mu_{\mathrm{x}}),$$

so that

$$\begin{pmatrix} U_i^o \\ U_i \end{pmatrix} \sim N_3\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \delta^t \\ \delta & \Omega \end{pmatrix}\right) \tag{3.1}$$

and $\delta^t = (\beta\sigma_x, \sigma_x)$. From the conditioning method discussed in the previous section and (3.1) we obtain the conditional distribution of $U_i \mid (U_i^o \geq -\tau)$, which will be called "extended skew-normal measurement error model", with density function given by

$$\varphi_2(z; \mu, \Omega) \frac{\Phi\left(\alpha_o + \lambda^t (z - \mu)\right)}{\Phi(\tau)}, \tag{3.2}$$

where λ and α_o are given by (1.4). In order to use (3.2) we do need some additional results which are presented next.

Theorem 3.1: *The skewness parameter, λ, and the parameter, α_o, of the extended skew-normal measurement error model are given by*

$$\lambda^t = \frac{\sigma_x b^t \Psi^{-1}}{(1 + b^t \Psi^{-1} b \sigma_x^2)^{\frac{1}{2}}} \quad \text{and} \quad \alpha_{\mathrm{o}} = \tau\left(1 + \mathrm{b}^{\mathrm{t}}\Psi^{-1}\mathrm{b}\sigma_{\mathrm{x}}^2\right)^{\frac{1}{2}}, \tag{3.3}$$

respectively, where $\Psi = diag\left(\sigma_e^2, \sigma_u^2\right)$ and $b^t = (\beta, 1)$.

Proof. The results follow from (1.4) after some tedious but simple algebraic manipulations and by noting that

$$\Omega^{-1} = \left(\Psi + \sigma_x^2 b b^t\right)^{-1} = \Psi^{-1} - \frac{\sigma_x^2 \Psi^{-1} b b^t \Psi^{-1}}{(1 + b^t \Psi^{-1} b \sigma_x^2)}.$$

In fact, from Theorem 3.1, we can write $\lambda^t = (\lambda_1, \lambda_2)\,, \alpha_o$ and Ψ in a different way which will be helpful in the implementation of the Bayesian procedure via Winbugs in the simulation section:

$$
\begin{aligned}
\lambda_1 &= \frac{\sigma_x \beta}{\sigma_e^2 \left(1+\sigma_x^2\left(\frac{\beta^2}{\sigma_e^2}+\frac{1}{\sigma_u^2}\right)\right)^{\frac{1}{2}}},\ \lambda_2 = \frac{\sigma_{\mathrm{x}}}{\sigma_{\mathrm{u}}^2 \left(1+\sigma_{\mathrm{x}}^2\left(\frac{\beta^2}{\sigma_{\mathrm{e}}^2}+\frac{1}{\sigma_{\mathrm{u}}^2}\right)\right)^{\frac{1}{2}}}, \\
\alpha_{\mathrm{o}} &= \tau \left(1+\sigma_x^2\left(\frac{\beta^2}{\sigma_e^2}+\frac{1}{\sigma_u^2}\right)\right)^{\frac{1}{2}}
\end{aligned}
$$

and

$$
\Omega^{-1} = \begin{pmatrix} \frac{1}{\sigma_e^2} - \lambda_1^2 & -\lambda_1\lambda_2 \\ -\lambda_1\lambda_2 & \frac{1}{\sigma_u^2} - \lambda_2^2 \end{pmatrix}.
$$

We end this section by making some remarks on the extended skew-normal measurement error model considered in the previous section:

- Assuming that $\tau = 0$ we obtain a strong asymmetry in x_i, that is , $x_i \geq \mu_x$. In this particular situation we obtain the skew-normal measurement for Z given by

$$
2\varphi_2\left(z;\mu,\Omega\right)\Phi\left(\lambda^t(z-\mu)\right), \tag{3.4}
$$

which is the result obtained in Corollary 1 by Arellano-Valle et al. (2005).

- Azzalini and Capitanio (1999) showed that (3.2) is quite closely approximated by (3.4), more explicitly, for each member of the four-parameter class (1.1) , there is a member of the three-parameter class (1.2) which is close to it as for numerical values of the density. This fact causes some problems to locate parameters. However, it works but in some cases larger chain were needed to get convergence in the simulation study via MCMC and it is not rosbust with respect to starting values We will be back to this point in the simulation section.

- Using the approach proposed in this paper it was possible to model the skewness parameters in terms of the original parameter avoiding the fomulation of new priors for them and the need of usual constrains to solve identification problems in the measurement errors models (see, Fuller´s book, 1987).

- The skew-normal model presents nice properties, but some strange behavior may arise when estimating the skewness parameter (see Azzalini, 1985; Liseo and Loperfido, 2002). The extended model considered in this section does not have this kind of problem since the skewness parameter only depends on the parameters involved in the model. Also, an addition information is provided when perfoming a default Bayesian analysis for this skew-normal measurement error model. This Bayesian approach is presented in the next section.

4 Bayesian inference

In this section, we consider the Bayesian approach to estimate the parameter $\theta = \left(\alpha, \beta, \mu_x, \sigma_x^2, \sigma_e^2, \sigma_u^2, \tau\right)$ with the following prior specifications:

1. $\tau \sim SN\left(0, \kappa, \lambda^*\right)$,
2. $\alpha \sim N\left(0, \sigma_\alpha^2\right)$,
3. $\beta \sim N\left(0, \sigma_\beta^2\right)$,
4. $\mu_x \sim N\left(0, \sigma_{\mu_x}^2\right)$,
5. $\frac{1}{\sigma_a^2} \sim Gamma\left(\gamma_a, \gamma_a\right),\ a = x, e, u,$

where $\lambda^*, \sigma_\alpha^2, \sigma_\beta^2, \sigma_{\mu_x}^2$, and γ_a, for $a = x, e, u$, are fixed hyperparameters.

Under the skew-normal measurement error model presented by (3.2)-(3.3) the log-likelihood function, denoted by $l\left(\theta\right)$, can be written as

$$\begin{aligned} l\left(\theta\right) \quad \propto \quad & -\frac{n}{2}\log\left(\left|\Omega\right|\right) - \frac{1}{2}\sum_{i=1}^{n}\left(z_i - \mu\right)^t \Omega^{-1}\left(z_i - \mu\right) \\ & + \sum_{i=1}^{n}\Phi\left(\alpha_o + \lambda^t\left(z_i - \mu\right)\right) - n\log\left(\Phi\left(\tau\right)\right), \end{aligned} \tag{4.1}$$

where z_i, μ and λ are defined in the previous sections.

For $\tau = \infty$, we obtain the usual likelihood considered by Fuller (1987) and $\tau = 0$ we obtain the likelihood given by Arellano-Valle et al. (2004) under the skew-normal distribution. Since the goal is to make easy the implementation of the Bayesian procedure, using the Winbugs software, in the next sections we will try to eliminate the third right expression of (4.1) by introducing the following truncated latent random variables :

$$T_i \mid T_i > 0 \sim N\left(\eta_i, 1\right),\ i = 1, \ldots, n,$$

where

$$\eta_i = \alpha_o + \frac{\sigma_x b^t \Psi^{-1}\left(z_i - \mu\right)}{\left(1 + b^t \Psi^{-1} b \sigma_x^2\right)^{\frac{1}{2}}}, \tag{4.2}$$

so that, the log-likelihood function based on the augmented data, $\left(Z, T\right)$, (see Tanner and Wong, 1987), denoted by $l_a\left(\theta\right)$, is given by

$$\begin{aligned} l_a\left(\theta\right) \quad \propto \quad & -\frac{n}{2}\log\left(\left|\Omega\right|\right) - \frac{1}{2}\sum_{i=1}^{n}\left(z_i - \mu\right)^t \Omega^{-1}\left(z_i - \mu\right) \\ & -\frac{1}{2}\sum_{i=1}^{n}\left(t_i - \eta_i\right)^2 - n\log\left(\Phi\left(\tau\right)\right). \end{aligned} \tag{4.3}$$

Unfortunately, the log-likelihood function given by (4.3) is not listed in the Winbug´s manual, so, the trick of create a new vector 'ones ´ will be used in the next section to compute the marginal posteriors of the components of θ.The latent variable (4.2) was also obtained by Arellano-Valle et al. (2005) by using the Henze stochastic representation (Azzalini, 1985).

5 A simulation study

In this section we briefly illustrate the behavior of the Bayesian procedure formulated in the previous section considering a data set of size, $n = 300$, generated according the measurement error model given by (2.1)-(2.3), but assuming that x_i follows a skew-normal distribution with skewness parameter $\lambda_x = 10$. For this generated sample we take $\mu_x = 5, \alpha = 1, \beta = 2$ and $\sigma_e^2 = \sigma_u^2 = \sigma_x^2 = 1$. For this simulation we assume proper and almost flat priors for the parameters according the previous specified priors ($\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_{\mu^*}^2 = 10^3$ and $\gamma_a = 0.01$), but considering a strong positive asymmetry on x, that is, $\kappa = 0$ which is equivalent to assume that $\tau = 0$. The augmented algorithm has been implemented using the one-zero trick mentioned in Winbugs´s manual and Bugs code are available from the author upon request. Unfortunately, we have some difficulties in estimating the parameter α, so, to solve this problem and to improve the convergence we introduce a new parameter, α^*, in the error model given by (2.1) resulting the following regression equation:

$$Y_i = \alpha^* + \beta(x_i - \mu_x) + e_i,$$

where $\alpha^* = \alpha + \beta\mu_x = 11$. A similar reparametrisation was suggested by Azzalini (1985) to obtain estimates from direct maximisation of the log-likelihood. The results are presented in Table 1.

Table 1 *Posterior summaries.*

parameter	mean	sd	2.5%	median	97.5%
α^*	10.95	0.6392	9.453	11.17	11.61
β	1.885	0.7839	1.031	1.619	3.757
μ_x	4.783	0.1236	4.555	4.779	5.045
σ_e^2	1.198	0.565	0.03232	1.39	1.847
σ_u^2	0.821	0.115	0.6211	0.8112	1.065
σ_x^2	1.322	0.3402	0.7006	1.296	2.045

Following the convergence criterions suggested in the Winbugs software, we use 17,798 iterates after burning 4,000 iterates to obtain the convergence. The estimates present a reasonable agreement with the real parameters for this particular generated data set and it call our attention the nice estimate of the parameter α^* . It was generated other data sets and the results are quite similar to the one presented in Table 1. If we not assume a strong asymmetry in x we can have some difficulties to obtain convergence as mentioned by Azzalini and Capitanio (1999), sect 4.2. That is, the presence of the parameter τ in conjuction with other parameters implies identifiable problems which could be removed using the profile log-likelihood as function of τ (see Azzalini and Capitanio, 1999, for more details). The reader can obtain more details about that in Capitanio et al. (2003).

6 An application: AIS data

In this section, we consider the Bayesian analysis of the AIS data included in the library sn for R (available for download at http://azzalini.stat.unipd.it/SN/index.html). We define the regression measurement error model relating SSF_i and $Bfat_i$ as follows:

$$\begin{aligned} SSF_i &= \alpha + \beta bfat_i + e_i, \\ Bfat_i &= bfat_i + u_i, \mathrm{i} = 1, \ldots, 202, \end{aligned} \tag{6.1}$$

where $bfat_i$ is the real unknown value of the body fat percentage of the i-th individual in the sample ,$Wt_{i,}$, and SSF_i is the the sum of skin folds. The error variables $e_i$and u_i are assumed to be independent realizations of the $N\left(0, \sigma_e^2\right)$ and $N\left(0, \sigma_u^2\right)$,respectively. A simple plot of the histogram of the observed $Bfat_i$ (see Figure 1) clearly indicates a strong right asymmetry, so we consider the extended measurement error model introduced in this paper. Assuming $\tau = 0$ and the same noninformative priors used in the previous section for the other parameters, Table 2 reports the posterior summaries of the parameters involved in the error model (6.1). We had seen that the behaviour of the posterior estimates is quite similar the one presented by Arellano-Valle et al. (2005). As mentioned by them, the naive estimator (mle) of β is 5.04, which is about 10% smaller than the mle, thus indicating clear attenuation due to measurement error.

Table 2 *Posterior summaries.*

parameter	mean	sd	2.5%	median	97.5%
α	-4.393	1.663	-7.772	-4.358	-1.198
β	5.434	0.1133	5.218	5.432	5.662
μ_x	6.624	0.4311	5.202	6.684	7.179
σ_e^2	7.19	6.13	1.14	6.156	25.03
σ_u^2	2.584	0.3593	1.808	2.59	3.28
σ_x^2	87.58	10.65	69.82	86.54	112.1

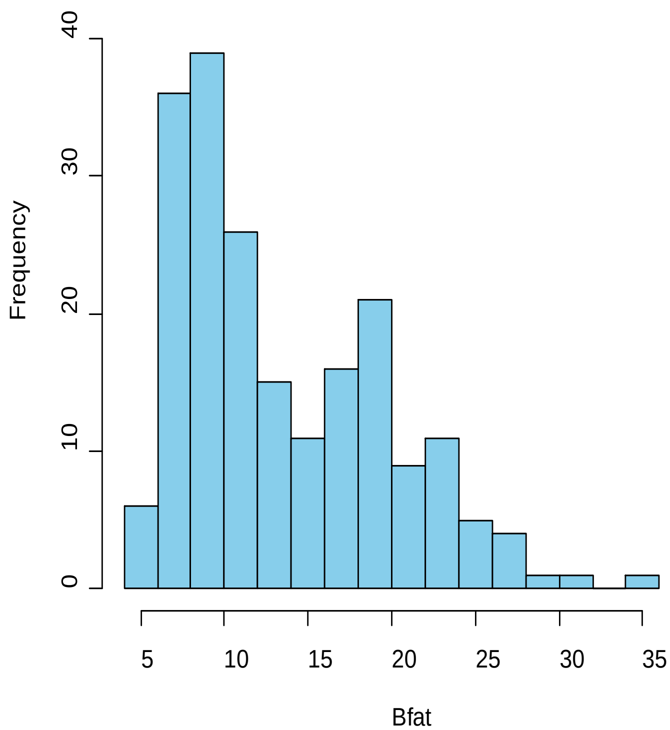

Figure 1 *Histogram for Bfat.*

7 Discussion

In the last years we have seen an increase of papers dealing with departures from normality, specially in terms of skewness. Among the many proposals to introduce asymmetry in the regression models with error in variables, we proposed in this paper a new skew- normal measurement error model assuming right skewness in the unobserved real value of the observable regression variable. The main idea was to use the conditioning method under constrains to obtain an extended skew-normal measurement error model which is similar to the one formulated by Arellano-Valle et al. (2003) when a strong asymmetry is considered. Inferential problems was presented from the Bayesian point of view by taking proper and flat priors via Winbugs. The usefulness of the proposed error model is investigated with a simulation study and real data analysis showing a good performance. The main advantage of the Bayesian approach is the possibility to measure the degree of belief that the true value of the explanatory variable is greater than its mean value. The presence of the additional parameter τ implied identifiable problems which could be removed by using the profile log-likelihood as function of τ as mentioned by Capitanio et al. (2003). This identification problem will be the subject of a future paper. If $\tau = 0$, that is, a strong asymmetry is assumed, the implementation of the Bayesian procedure can be easily done using Winbugs´s software and the zero-one trick without convergence problems. One important aspect of the approach used in this paper, when comparing with Arellano-Valle et al.´s approach (2005), is the Bayesian results obtained are not invariant in terms of skewness on x. We end this paper by calling the attention on the flexibility of the proposed skew -normal error model to avoiding the usual constraints on the regression measurement errors and the usefulness of the Bayesian approach.

Acknowledgements

The author would like to thank Heleno Bolfarine and the referee for their comments and suggestions to improve the reading of this paper. This research was supported by Conselho Nacional de Pesquisa- CNPq- Grant no.300852/83-5.

(Received November, 2005. Accepted May, 2006.)

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